

Pion-Pion Phase-Shifts and the Value of Quark-Antiquark Condensate in the Chiral Limit

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We use low energy pion-pion phase-shifts in order to make distinction between the alternatives for the value of the quark-antiquark condensate B_0 in the chiral limit. We will consider the amplitude up to and including $\mathcal{O}(p^4)$ contributions within the Standard and Generalized Chiral Perturbation Theory frameworks. They are unitarized by means of Padé approximants in order to fit experimental phase-shifts in the resonance region. As the best fits correspond to $\alpha = \beta = 1$, we conclude that pion-pion phase-shift analysis favors the standard ChPT scenario, which assumes just one, large leading order parameter $\langle \bar{q}q \rangle_0$.

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I. INTRODUCTION

Strong interaction phenomena at low energies are very constrained by chiral symmetry and by the structure of the Quantum Chromodynamics (QCD) ground state, which involves the quark and the gluon condensates. Massless QCD ($m_u = m_d = m_s = 0$) is symmetric under the group $SU(3)_L \times SU(3)_R$ that is spontaneously broken to $SU(3)_V$, what implies the existence of eight Goldstone bosons coupled to the corresponding conserved axial quark currents. The coupling strength is related to the pseudo-scalar meson decay constant F , which in the chiral limit is of order 100 MeV, much smaller than typical hadron masses, say 1 GeV.

One may ask what is the size of the quark condensate parameter

$$B_0 = -\frac{\langle \bar{q}q \rangle_0}{F^2}.$$

Lattice calculations allow one to access this information, which is needed, for instance, in the applications of QCD sum rules. Another phenomenological approach that refers to this quantity is Chiral Perturbation Theory (ChPT) [1] the only theory for low energy QCD. ChPT Lagrangian for meson processes is a series of terms of different orders in the covariant derivatives of Goldstone fields and their masses. However, the order in the external momenta p of the contribution of each term of the Lagrangian to a given process depends on how big B_0 is, as will be explained in the sequence. In the standard ChPT, B_0 is assumed to be as large as $\Lambda_H \sim 1$ GeV. On the other hand, by considering the possibility of a low value for the quark condensate, some authors [2] have developed a program named Generalized Chiral Perturbation Theory (GChPT). As a new feature, the series of terms in the Lagrangian keeps B_0 as an expansion parameter of $\mathcal{O}(p)$, as well as the quark masses, which are in turn accounted for as $\mathcal{O}(p^2)$ in the standard case.

Our motivation in the present exercise is that, as it is well known, the values for the pion-pion S-wave scattering length within the two approaches differ by 30%, being the one from the generalized approach closer to the experimental value than in the standard case. Here we try to look at another phenomenological consequence of these different frameworks. The standard and generalized scenarios can be analyzed by a global fit of pion-pion phase-shifts. In this case, in order to access the resonance region of pion-pion scattering using ChPT, one may wish to use Padé approximants, as e.g. advocated in Ref. [3], giving rise to the so called inverse amplitude method (IAM).

In the next section we present the ChPT pion-pion scattering amplitude and the corresponding partial-waves. In section III IAM is described and the fits to experimental data are shown. The last section includes our conclusion and final remarks.

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II. STANDARD AND GENERALIZED CHPT

In the standard ChPT [1] the leading contribution to low energy pion-pion scattering reproduces the soft-pion amplitude obtained by Weinberg [4], given by

$$A(s, t, u) = \frac{1}{F^2} (s - 2\hat{m}B_0), \quad (1)$$

where $\hat{m} = (m_u + m_d)/2$. The leading order value of the pion mass in the standard ChPT is denoted by M , with

$$M^2 = 2\hat{m}B_0.$$

On the other hand, in GChPT one gets the same structure as (1), with B_0 replaced by

$$B = B_0 + 2m_s Z_0^S,$$

so that (1) reads

$$A(s, t, u) = \frac{1}{F^2} \left(s - M^2 - r \frac{M^4}{B_0^2} Z_0^S \right). \quad (2)$$

The new low energy constant (LEC) introduced here is Z_0^S , which is the coefficient of $[\text{Tr}(U^\dagger \chi + \chi^\dagger U)]^2$ in the term of order p^2 of the GChPT Lagrangian, whereas in the standard case this term would be actually accounted for as $\mathcal{O}(p^4)$. The expression (2) also depends on the ratio $r = m_s/\hat{m}$, where m_s is the strange quark mass.

These differences between the two approaches are more important when including loop corrections to the total amplitude. In this case, some terms that are considered to be of orders p^6 and p^8 in the standard approach contribute to the one loop result. The general form of the amplitude up to one loop can be written as [5]

$$\begin{aligned} A(s, t, u) = & \frac{\beta}{F_\pi^2} \left(s - \frac{4}{3} M_\pi^2 \right) + \alpha \frac{M_\pi^2}{3F_\pi^2} \\ & + \frac{1}{F_\pi^4} (s - 2M_\pi^2)^2 \lambda_1 + \frac{1}{F_\pi^4} \left[(t - 2M_\pi^2)^2 + (u - 2M_\pi^2)^2 \right] \lambda_2 \\ & + \frac{1}{6F_\pi^4} \left\{ \left[4 \left[\beta \left(s - \frac{4}{3} M_\pi^2 \right) + \frac{5}{6} \alpha M_\pi^2 \right]^2 - \left[\beta \left(s - \frac{4}{3} M_\pi^2 \right) - \frac{2}{3} \alpha M_\pi^2 \right]^2 \right] \bar{J}(s) \right. \\ & \left. + \left[\frac{1}{2} \left[3 \left(\beta \left(t - \frac{4}{3} M_\pi^2 \right) - \frac{2}{3} \alpha M_\pi^2 \right)^2 + \beta^2 (s - u) (t - 4M_\pi^2) \right] \bar{J}(t) + (t \leftrightarrow u) \right] \right\}, \end{aligned} \quad (3)$$

where $\bar{J}(x)$ is the loop integral, given by

$$16\pi^2 \bar{J}(x) = 2 + \sigma(x) \ln \frac{\sigma(x) - 1}{\sigma(x) + 1}, \quad \sigma(x) = \sqrt{\frac{x - 4M_\pi^2}{x}}.$$

This structure is equivalent to the expression in Eq. 2.1 of Ref. [6], obtained long ago by Sá Borges in the context of the Unitarization Program of Current Algebra (UPCA), if one takes α and β equal to one. Let us mention that UPCA polynomials in energy are different from those in eq. (3) because their coefficients are collections of subtraction constants inherent to the dispersion relation technique, whereas in ChPT they come from combinations of LECs and chiral logarithms.

The isospin defined amplitudes T_I for $I = 0, 1$ and 2 are

$$\begin{aligned} T_0(s, t) &= 3A(s, t, u) + A(t, s, u) + A(u, t, s), \\ T_1(s, t) &= A(t, s, u) - A(u, t, s), \\ T_2(s, t) &= A(t, s, u) + A(u, t, s), \end{aligned}$$

which are expanded in partial-wave amplitudes, as

$$T_I(s, t) = \sum_{\ell} (2\ell + 1) t_{\ell I}(s) P_{\ell}(\cos \theta),$$

where P_ℓ are the Legendre polynomials. In the following we omit the label ℓ , because we just deal with S-wave ($I = 0, 2$) and P-wave ($I = 1$).

Elastic unitarity implies that, for $16m_\pi^2 \geq s \geq 4m_\pi^2$,

$$\text{Im } t_I(s) = \frac{1}{16\pi} \sigma(s) |t_I(s)|^2, \quad (4)$$

which can be solved yielding

$$t_I(s) = \frac{16\pi}{\sigma(s)} e^{i\delta_I(s)} \sin \delta_I(s), \quad (5)$$

where $\delta_I(s)$ are the real phase-shifts.

III. FITTING PION-PION PHASE-SHIFTS

Our strategy is to disentangle between standard and generalized versions of ChPT by fitting experimental phase-shifts. Since ChPT partial-wave amplitudes respect eq. (4) only approximately they must be unitarized. In order to do this we follow the procedure of the inverse amplitude method (IAM) [3], that amounts to writing

$$\tilde{t}_I(s) = \frac{t_I^{(2)}(s)}{1 - t_I^{(4)}(s)/t_I^{(2)}(s)}, \quad I = 0, 1 \text{ and } 2, \quad (6)$$

where $t^{(2)}$ and $t^{(4)}$ are partial-wave projections of respectively $\mathcal{O}(p^2)$ and $\mathcal{O}(p^4)$ terms in eq. (3).

This expression allows one to access the resonance region for pion-pion scattering if one fixes the free parameters λ_1 and λ_2 by fitting the phase-shifts defined in eq. (5) to experimental data. We recall that these parameters are collections of LECs. In ChPT, the LECs are not related to the explicit symmetry breaking parameter and have to be obtained phenomenologically. The traditional way to do that is to extract them from $K_{\ell 4}$ decay and from D-wave scattering length, for instance. Here the LECs corresponding to the parameters obtained from our fits are not expected to be exactly the ones found in literature, but not very far from these, since the low energy behaviour of IAM amplitudes are approximately the same as in the non-corrected ones.

In the general amplitude given by (3), besides the constants λ_1 and λ_2 , that will be used to fit the phase-shifts, we have also the constants α and β , which can be written in terms of several LECs, such as B_0 and Z_0^S as well as other ones coming from higher orders and loop contributions. In the standard case, α and β are kept very close to one, while in GChPT they can change within a much wider range. The allowed values of these constants [5] depend on the ratio $r = m_s/\hat{m}$ and are very close to the corresponding ones of the standard ChPT for large values of r , namely greater than 25. Particularly, for a reasonable value, say $r = 10$, the allowed ranges are $2.2 < \alpha < 3.3$ and $1.12 < \beta < 1.24$ approximately. Regardless the value of r , α can vary from 1 to 4 and β , up to 1.3.

For each fixed choice of α and β within the ranges mentioned right above, we perform the fit of P-wave phase-shift, which determines the combination $\lambda_2 - \lambda_1$. This value is then introduced in the expression of $I = 0$ S-wave in order to determine the remaining parameter by fitting experimental data. $I = 2$ amplitude does not depend on any new parameter. For each set of α and β , we could evaluate the quality of the fit, by considering the related χ^2 .

Fig. 1 shows the χ^2 of the fits for several choices of α and β . One can see that, for a given value of β , the χ^2 is not considerably modified within the range of α values allowed according to Ref. [5]. The quality of fits is very dependent on β and the best fit corresponds to the bullet in the figure ($\alpha = 1$ and $\beta = 1$). According to this analysis, Figs. 2 and 3 present the resulting phase-shifts¹ of respectively P- and S-waves, for only one choice of α for each value of β .

The table presents the values of parameters λ_1 and λ_2 for P1- and S0-wave fits and the value of the related χ^2 's. Clearly the fits get poorer for large values of β . It means that, for $r \simeq 10$ the standard ChPT approach is more suitable than GChPT to fit $\pi\pi$ phase-shifts in a wide energy range. Nevertheless if one accepts a large enough value for r , then the two approaches are indistinguishable within this method.

¹Experimental data for P-wave were taken from Ref. [10], for $I = 0$ S-wave from Ref. [10] (squares) and Ref. [11] (triangles) and for $I = 2$ S-wave from Ref. [12].

IV. CONCLUSION

As a summary, we have presented a study motivated by discrepancies between the standard and generalized ChPT predictions for S-wave pion-pion scattering length. Our concern was to verify whether pion-pion phase-shifts could distinguish between the large and low value of the quark condensate. In order to achieve it, we have fitted experimental data using the expressions following from the two scenarios. Our work favors the standard approach of ChPT, which assumes a large vacuum expectation value for the quark-antiquark condensate operator.

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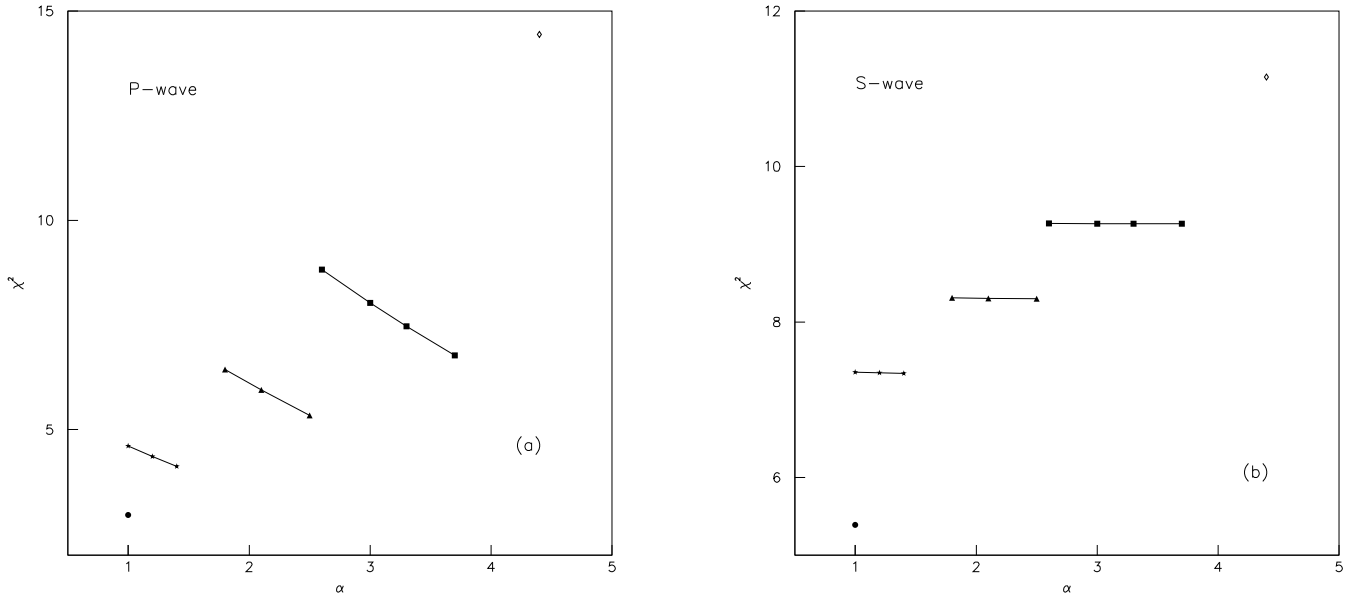


FIG. 1. χ^2 of (a) P-wave and (b) and S-wave fits to pion-pion phase-shifts, as a function of α (adimensional); each symbol corresponds to a singular value of β .

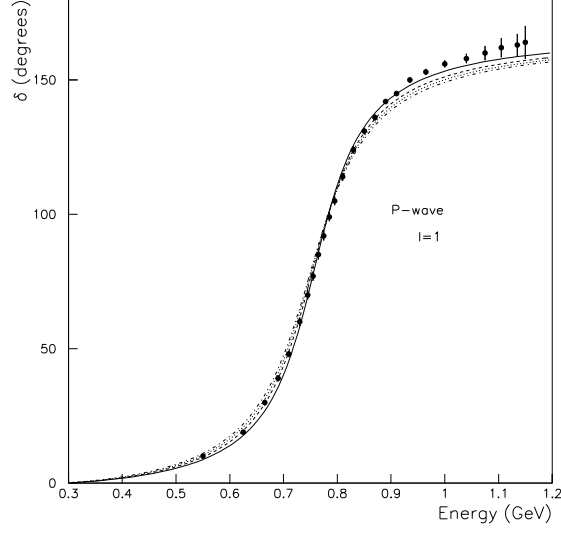


FIG. 2. P-wave pion-pion phase-shift, in degrees, as a function of center of mass energy, in GeV; experimental data (bullets) and results from fits, for α and β as given in the Table.

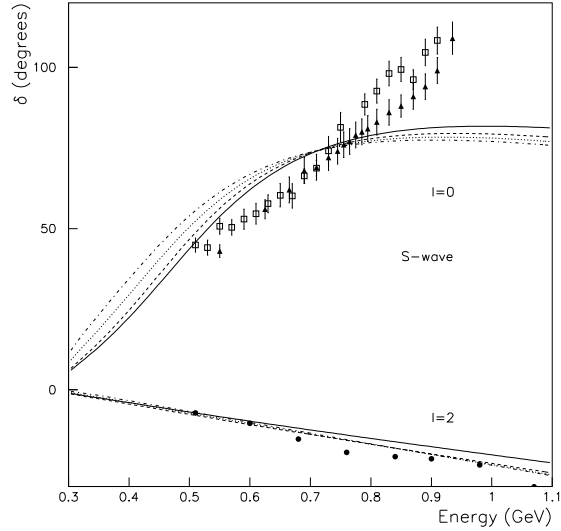


FIG. 3. S-wave pion-pion phase-shift, in degrees, as a function of center of mass energy, in GeV; experimental data for $I = 0$ (squares and triangles) and $I = 2$ (bullets) and results from fits of $I = 0$ wave, for α and β as given in the Table.

α	β	line type	$\lambda_1 \times 10^3$	$\lambda_2 \times 10^2$	χ^2 (P1)	χ^2 (S0)
1.0	1.0	solid	-3.024	1.1702	2.96	5.39
1.0	1.1	dashed	-3.268	1.2971	4.61	7.36
2.6	1.2	dotted	-3.858	1.4999	8.82	9.27
4.4	1.3	dot-dashed	-4.562	1.7235	14.4	11.2

TABLE I. Fitted parameters λ_1 and λ_2 and χ^2 of the fits, corresponding to the various curves in Figs. 1 and 2. The χ^2 of the N -point fit is given, as usual, by the sum of deviations from data squared, weighted by error bars squared, and divided by $(N - 1)$.